# Uses and Abuses of Non-parametric Statistics in Medical Research

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Sponsored by the Clinical and Translational Science Institute (CTSI) and the Department of Population Health / Division of Biostatistics

# Speaker Disclosure

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# Parametric Statistics

- Body of Statistical Methods Based on an Assumed Model for the Underlying Population from which the Data was T /Lbs/>ledSe/ying

# Example of Parametric Procedures

- Classical t-test assumes normal populations
- Analysis of Variance assume normal populations
- Logistic regress assumes binomial population
- Pearson's correlation coefficient assumes bivariate normal

# Simple Example

| Patient | Pre Score X | Post Score Y | Change |
|---|---|---|---|
| | | | |
| | | | |
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Data

1.03 -0.15 1.02 0.43 0.62 0.59 0.49 0.02 0.01

# Sign Test

- Count number of positive differences s=8
- If null hypothesis is true the number of positive differences is like flipping a fair coin n times so

P-value =Pr[S∅s|n,1/2]  (one sided)

= 2xSmaller of {Pr[S∅s|n,1/2 ], Pr[S  s|n,1/2 ]}

- P value= Pr[S∅8|9,1/2 ]=0.019
- Hence Hamilton score decreased by tranquilizer

# Wilcoxon Sign Rank Test

| Patient | d=X-Y | Rank \|d\| |
|---|---|---|
| 1 | 1.03 | 9 |
| 2 | -0.15 | **3** |
| 3 | 1.02 | 8 |
| 4 | 0.43 | 4 |
| 5 | 0.62 | 7 |
| 6 | 0.59 | 6 |
| 7 | 0.49 | 5 |
| 8 | 0.02 | 2 |
| 9 | 0.01 | 1 |

# Wilcoxon Sign Rank Test

- Add up ranks associated with the positive (or negative) differences $T^+$ ($T^{-}$)
- Note sum of ranks =n(n+1)/2
- Under $H_o$ + and – values should be mixed so $T^+$ should

# Calculations

- $n(n+1)/2=9x10/2=45$
- $T^{-}=3$  $T^{+}=45-3=42$
- P-value from table p=0.006
-

# Why Nonparametrics?

6. Provides simple tests for complicated hypotheses
   - Example k sample

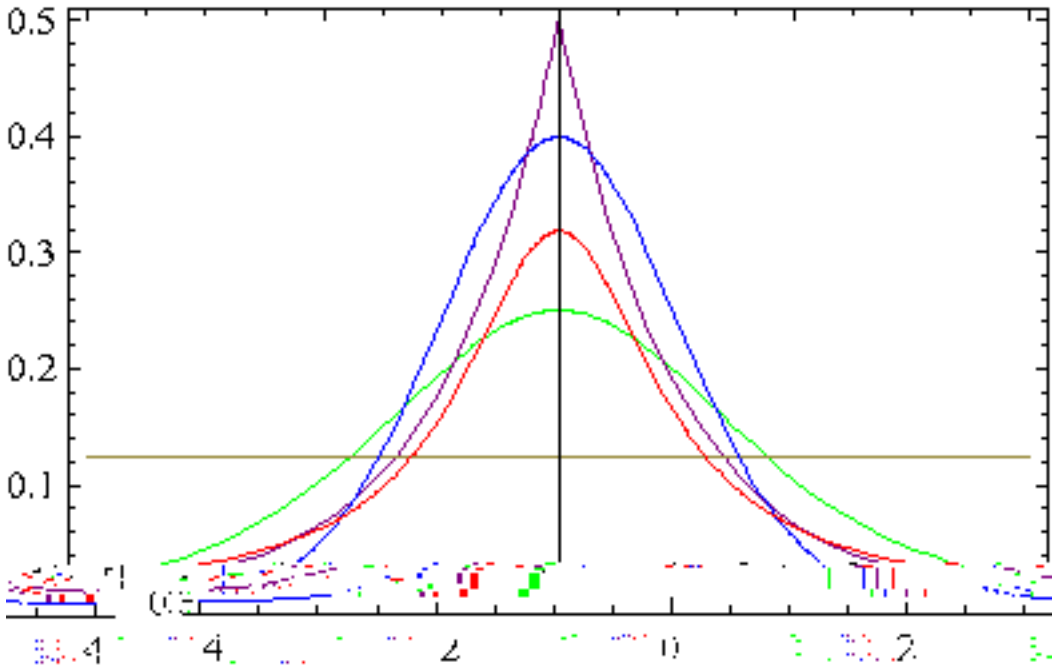
$H_o$: $\sigma_1=\sigma_2=. . . =\sigma_k$ versus

$H_o$: at least one $\sigma_j$ is different

Parametric: Analysis of VT.42 Tml1T0 1 1o303.06 Tm(j )TjE

# Why Nonparametrics?

7. Exact confidence intervals are available
8. Gives protection against outliers
9. Software for nonparametric methods is available in most packages
10. When software is not available replace data by ranks in normal theory software




| | Normal | Uniform | Logistic | Double Exponential | Cauchy |
|---|---|---|---|---|---|
| e(sign test,paired t test) | .637 | .333 | .822 | 2.00 | |
| e(Sign Rank, paired t) | .955 | 1.000 | 1.097 | 1.50 | |

# One or Paired sample tests

- Location
  - Parametric test:  t-test
  - Nonparametric tests
    - Sign Test  (No Assumptions)
    - Wilcoxon Sign Rank Test
      - Works best for symmetric populations
    - Confidence intervals based on Hodges-Lehman statistic

# Two Independent Samples Tests for equal variance

- Parametric test (assumes normality)
  - F test
- Non parametric tests
  - Ansari-Bradley test (assumes equal medians)
  1. Rank Data in pooled sample
  2. Assign scores 1 2 3 4 5 5 4 3 2 1 (n=10)
  3. Add up scores for 1st sample and reject if too small or too large
  - Miller's Jackknife test if unequal medians

# One way layout
# k=3 or more independent groups

- Parametric—(assumes normality, equal variances) Analysis of Variance for case 1
- Nonparametric Model $X_{ij}=\chi 2\vartheta_j+E_{ij}$, $i=1,\ldots,n_j$, $j=1,\ldots,k$
  - Case 1 $H_A$:[$\vartheta_1, \ldots, \vartheta_k$ not all equal]
    - Kruskal-Wallis test
  - Case 2 $H_A$:[$\vartheta_1 \quad \vartheta_2 \quad \ldots \quad \vartheta_k$ with at least one strict inequality]
    - Jonckheere-Terpstra Test

# One way layout
# k=3 or more independent groups

- Nonparametric Model $X_{ij}=\chi 2\vartheta_j+E_{ij}$, $i=1,\ldots,n_j$, $j=1,\ldots,k$
  - Case 3
  - $H_A$:[$\vartheta_1 \quad \vartheta_2 \quad \ldots \quad \vartheta_{p-1} \quad \vartheta_p \quad \vartheta_{p+1} \varnothing \ldots \varnothing \vartheta_k$ with at least one strict inequality]
    - Mack-Wolfe test

# Measures of Association

- Parametric—Pearson's Product moment correlation coefficient, R. -1 R +1
  - Measures strength of linear association when data is bivariate normal -1 R +1
  - When the marginals are not normal you can have perfect association and an R ∏ ±1

# Measures of Association Nonparametric

- Spearman's rho—Usual correlation coefficient using ranks
- Kendall's tau

  Pairs $(X_i,Y_i)$, $(X_j,Y_j)$ are

  concordant if $(X_i-Y_i)(X_j-Y_j) >0$;

  discordant if $(X_i-Y_i)(X_j-Y_j) <0$

  Tau =(#concordant-#discordant pairs)/(n(n-1)/2)

- Both measures are
  - Between -1,+1
  - Equal to 0 if X and Y are independent
  - Equal to +/-1 if Y=a+bX

# Remarks

- Most of these methods are in most

# Summary

- Nonparametric statistics provide a viable alternative to normal theory statistics
- Nonparametric statistics are particularly useful when
  - The sample size is small
  - You want protection from outliers
  - You have ordinal data

# Summary

Nonparametric statistics are particularly useful when

- You are modeling association between variables
- You are doing tests with ordered or umbrella alternative
- You have survival data

# Resources

- The Clinical and Translation Science Institute (CTSI) supports education, collaboration, and research in clinical and translational science: www.ctsi.mcw.edu
- The Biostatistics Consulting Service provides comprehensive statistical support

  http://www.mcw.edu/biostatsconsult.htm