A Complex fMRI Activation Model With a Temporally Varying Phase

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Abstract

Recently Rowe and Logan (2004) introduced a complex fMRI activation model in which multiple regressors were allowed, hypothesis tests were formulated in terms of contrasts, and the phase was directly modeled as a fixed unknown quantity which

where here $vec(\cdot)$ is used to denote an n dimensional vector whose t^{th} element is given by its scalar argument and $y_M = vec\left(\sqrt{y_{Rt}^2 + y_{It}^2}\right)$.

The maximum likelihood estimates under the constrained null hypothesis $H_0 : C\beta = 0$ are similarly derived in the appendix and given by

$$
\begin{aligned}
\tilde{\theta}_t &= \tan^{-1}\left(\frac{y_{It}}{y_{Rt}}\right), \quad t = 1,\dots,n \\
\tilde{\beta} &= \Psi\hat{\beta}, \\
\tilde{\sigma}^2 &= \frac{1}{2n}\left[y - \begin{pmatrix} \tilde{A}_1 X\tilde{\beta} \\ \tilde{A}_2 X\tilde{\beta} \end{pmatrix}\right]' \left[y - \begin{pmatrix} \tilde{A}_1 X\tilde{\beta} \\ \tilde{A}_2 X\tilde{\beta} \end{pmatrix}\right] \\
\Psi &= I_{q+1} - (X'X)^{-1}C'[C(X'X)^{-1}C']^{-1}C \ , \qquad (2.4)
\end{aligned}
$$

where $\tilde{A}_1$ and $\tilde{A}_2$ are diagonal matrices with $\cos\tilde{\theta}_t$ and $\sin\tilde{\theta}_t$ as the t^{th} diagonal element. The restricted regression coe cients can also be shown to be equivalent to the magnitude-only model because the multiplicative factor Ψ is identical in both cases.

2.2 Activation Statistics

The likelihood ratio statistic in Equation A.3 with some algebra can be written as

$$
F = \frac{(n-q-1)}{r}\left(\lambda^{-1/n} - 1\right) = \frac{(n-q-1)}{r}\frac{\hat{\beta}'C'[C(X'X)^{-1}C']^{-1}C\hat{\beta}}{2n\hat{\sigma}^2} \ . \qquad (2.5)
$$

Note that since

$$
2n\hat{\sigma}^2 = \left[y - \begin{pmatrix} \hat{A}_1 X\hat{\beta} \\ \hat{A}_2 X\hat{\beta} \end{pmatrix}\right]' \left[y - \left(\hat{A}_1 \right.\right.
$$

where r is the full row rank of C. Otherwise, one might use the Ricean distribution [4, 8] to derive the proper distribution of the F statistic. In either case, the estimates of β and the likelihood ratio test depend only on the magnitude data.

Note from (2.6) that the maximum likelihood estimate of σ^2 from the dynamic phase complex model is inconsistent, since it can be shown as follows that its expected value does not converge in probability or tend to its populaton value as the sample size tends to infinity

$$\begin{aligned} E\left(\hat{\sigma^2}\right) &= \frac{1}{2n}E\left\{\sum_{t=1}^{n}[y_{Mt} - x_t'\hat{\beta}]^2\right\} \\ &= \frac{1}{2n}\left\{(n-q-1)\sigma^2\right\} \\ &\xrightarrow{p} \frac{\sigma^2}{2}. \end{aligned}$$

An unbiased estimate of the variance can be obtained by simply using the unbiased estimate of the variance from the magnitude-only model.

3 Conclusions

A generalization of the constant phase complex activation fMRI model of Rowe and Logan (2004) was developed, where the phase angle is allowed to vary at each time point. It is shown that the estimated regression coe cients and the likelihood ratio F statistic for this dynamic phase complex fMRI model are equivalent to those in the usual magnitude-only model. It is also seen that the maximum likelihood estimate of the variance in this model is not consistent, but that a consistent variance estimate is obtained by simply using the magnitude-only unbiased variance estimate. Therefore, inference on task-related magnitude activation which is equivalent to that of the magnitude-only model can be derived directly from the dynamic phase complex model.

A Generalized Likelihood Ratio Test

A.1 Complex Model with θ_t

Unrestricted MLE's

Maximizing this likelihood with respect to the parameters is the same as maximizing the logarithm of the likelihood with respect to the parameters and yields

$$\left.\frac{\partial LL}{\partial \beta}\right|_{\beta=\hat{\beta},\theta=\hat{\theta},\sigma}$$

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